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Space-time evolution of the $\lambda\phi^4$ model: classical and quantum aspects

Fábio L. Braghin*and Fernando S. Navarra†

Instituto de Física da Universidade de São Paulo

C.P. 66.318, C.E.P. 05315-970, São Paulo, Brasil

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Abstract

A time dependent variational approach is used to derive the equations of motion for the $\lambda\phi^4$ model. The simultaneous evolution of the quantum fluctuations and of the classical part of the field is considered in a lattice of 1+1 dimensions. Different initial conditions corresponding to non equilibrium situations are considered and evolved in time. Such high energy localized configurations expand in the lattice by “bumps” which may change with time. The quantum fluctuations make the peaks be smoother and the expansion faster.

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*email:braghin@if.usp.br

†email:navarra@if.usp.br

1 Introduction

In relativistic heavy ion collisions, such as those performed now at RHIC and CERN, very dense and nearly baryon free systems are formed in the central rapidity region. These excited systems decay and produce a large number of secondary particles. In view of the large densities and multiplicities, with or without quark gluon plasma formation, it is very likely that these intermediate systems will exhibit some collective behavior. It is therefore reasonable to treat their expansion with hydrodynamics [1, 2, 3] and it is interesting to investigate the connection between the hydro-dynamical and the field theoretical approaches, see for example [4]. In fact, the fluid approach has been taken so seriously that irregularities in the hydrodynamic flow pattern have been proposed as possible signals of a deconfined phase [5].

Given the physical appeal of hydrodynamics, many of its aspects and underlying assumptions have been investigated during the last ten years. Some of the studied topics are thermalization, initial conditions for the fluid expansion, the equation of state (and possible phase transitions), the freeze out mechanism and final state interactions. The direct correspondence of the hydro-dynamical description with the field theoretical one is thus highly appropriate.

In this context, at least two questions arise which have not been addressed so far and are discussed in the present article. The first is: i) which is the role played by the quantum field nature of this dense matter in the expansion? In other words, which mistake are we doing when we use classical hydrodynamics and neglect quantum effects? The second question is: ii) if the expanding matter is merged in a condensate, which is the effect of this condensate on the expansion?

Whereas solving these equations is technically very complicated in full hydrodynamics, the interplay between classical background and quantum fluctuations can be studied in simpler scenarios. In this work we discuss the space-time evolution of the $\lambda\phi^4$ model in $1+1$ dimensions at zero temperature. Solving non-perturbatively and self-consistently the equations of motion of the theory, we can follow the expansion of a highly energetic system composed by a self interacting scalar field. In particular, we can separately investigate the classical and quantum components of the system and estimate the role played by the condensate during the expansion.

In spite of the enormous differences between this simple model and the realistic descriptions of the “fireballs” formed at RHIC, we believe that our study can be of interest and give us some relevant insights into the real physical situation.

Due to the extreme complexity of realistic theories, such as QCD, one usually is obliged to consider

effective models which respect the major properties of the fundamental theory. The $\lambda\phi^4$ model shares with QCD the properties of asymptotic freedom and spontaneous symmetry breaking (SSB) in the frame of the Gaussian approximation. Due to the SSB, the field is decomposed into a classical and a quantum part. At the same time it may also represent the mesonic sector of the linear sigma model, which is often used to describe the dynamics of a gas of pions. In the context of cosmology the scalar field of the model may be considered as the relevant degree of freedom for inflationary models [6].

Some progress has been achieved in the last decade in understanding the dynamics of the $\lambda\phi^4$ model. In particular, lattice calculations of the time evolution of out of equilibrium configurations, both at zero and finite temperatures, in 1+1 and 3+1 dimensions in continuous and discretized space-times have been performed [7, 8, 9, 10, 11, 12].

In this work the $\lambda\phi^4$ will be investigated in the Gaussian approach with the formalism discussed in Refs. [9]. It is suitable for non homogeneous configurations of the fluctuations (the quantum part of the field) as well as of the condensate (as the classical part of the field $\bar{\phi}(t)$ will be called). They can be thought of as two interacting “liquids” with continuous energy transfer and this view is under investigation. The text is organized as follows. In section II the Gaussian formalism for out of equilibrium systems is briefly outlined and the equations of movement are derived. In section III we discuss the numerical method employed here and developed in [9]. The numerical results are presented in section IV for several different out of equilibrium situations and values of the coupling constant. The results and some perspectives are summarized in the final section.

2 Time dependent Gaussian approximation to $\lambda\phi^4$ model

The Hamiltonian density for a scalar field ϕ with bare mass m_0^2 and coupling constant λ is:

$$H = \frac{1}{2} \left(\pi^2(\vec{x}) + (\nabla\phi(\vec{x}))^2 + m_0^2\phi^2(\vec{x}) + \frac{\lambda}{12}\phi^4(\vec{x}) \right), \quad (1)$$

where the action of operators ϕ and π in functional Schroedinger representation over a wave functional $\Psi[\phi(\vec{x})] = \langle \phi(\mathbf{x}) | \Psi[\phi] \rangle$ is given by:

$$\begin{aligned} \hat{\phi}|\Psi[\phi(\vec{x})]\rangle &= \phi(\vec{x})|\Psi[\phi(\vec{x})]\rangle, \\ \hat{\pi}|\Psi[\phi(\vec{x})]\rangle &= -i\delta/\delta\phi(\vec{x})|\Psi[\phi(\vec{x})]\rangle. \end{aligned} \quad (2)$$

For our variational calculations analyzed below, in the Schroedinger picture the wave functional evolves like the Schroedinger equation

$$i\frac{\partial}{\partial t}\Psi[\phi(\vec{x})] = H\Psi[\phi(\vec{x})]. \quad (3)$$

This is, thus, a non covariant formalism suitable for time dependent problems.

In the Gaussian approximation at zero temperature Ψ is parametrized by:

$$\Psi[\phi(\vec{x})] = N \exp \left\{ -\frac{1}{4} \int d\vec{x} d\vec{y} \delta\phi(\vec{x}) \left(G^{-1}(\vec{x}, \vec{y}) + i\Sigma(\vec{x}, \vec{y}) \right) \delta\phi(\vec{y}) + i \int d\vec{x} \bar{\pi}(\vec{x}) \delta\phi(\vec{x}) \right\}, \quad (4)$$

Where $\delta\phi(\vec{x}, t) = \phi(\vec{x}) - \bar{\phi}(\vec{x}, t)$; the normalization is N , the variational parameters are the condensate $\bar{\phi}(\vec{x}, t) = \langle \Psi | \phi | \Psi \rangle$ and its conjugated variable $\bar{\pi}(\vec{x}, t) = \langle \Psi | \pi | \Psi \rangle$; quantum fluctuations are represented by the width of the Gaussian $G(\vec{x}, \vec{y}, t) = \langle \Psi | \phi(\vec{x}) \phi(\vec{y}) | \Psi \rangle$ and its conjugate variable $\Sigma(\vec{x}, \vec{y}, t)$.

In variational time dependent calculations we have to choose an action to be minimized in order to obtain the equations of motion. We take the well known Dirac action from [13]:

$$I = \int dt \langle \Psi | \left(i\frac{\partial}{\partial t} - \hat{H} \right) | \Psi \rangle. \quad (5)$$

In order to calculate it, we take the mean value of an operator \hat{O} given by:

$$\langle \Psi | \hat{O} | \Psi \rangle = \int \mathcal{D}[\phi] \Psi^* \hat{O} \Psi \quad (6)$$

By means of this average procedure the energy density $\rho = \langle \Psi | H | \Psi \rangle$ can be calculated. Its temporal evolution will be investigated in section 4.

Variations with respect to the variational parameters and their conjugated yield the following equations of motion (any repeated spatial index means integration over that variable):

$$\begin{aligned} \frac{\delta I}{\delta \Sigma(\vec{x}, \vec{y})} &\rightarrow \partial_t G(\vec{x}, \vec{y}) = 2(G(\vec{x}, \vec{z})\Sigma(\vec{z}, \vec{y}) + \Sigma(\vec{x}, \vec{z})G(\vec{z}, \vec{y})), \\ \frac{\delta I}{\delta G(\vec{x}, \vec{y})} &\rightarrow \partial_t \Sigma(\vec{x}, \vec{y}) = \left(2\Sigma(\vec{x}, \vec{z})\Sigma(\vec{z}, \vec{y}) - \frac{1}{8}G^{-1}(\vec{x}, \vec{z})G^{-1}(\vec{z}, \vec{y}) \right) + \left(\frac{\Gamma(\vec{x}, \vec{y})}{2} + \frac{\lambda}{2}\bar{\phi}(\vec{x})^2 \right), \\ \frac{\delta I}{\delta \bar{\pi}(\vec{x})} &\rightarrow \partial_t \bar{\phi}(\vec{x}) = -\bar{\pi}(\vec{x}), \\ \frac{\delta I}{\delta \bar{\phi}(\vec{x})} &\rightarrow \partial_t \bar{\pi}(\vec{x}) = \Gamma(\vec{x}, \vec{y})\bar{\phi}(\vec{y}) + \frac{b}{6}\bar{\phi}^2(\vec{x}), \end{aligned} \quad (7)$$

Where $\Gamma(\vec{x}, \vec{y}) = -\Delta + \left(m_0^2 + \frac{\lambda}{2}G(\vec{x}, \vec{x}) \right) \delta(\vec{x} - \vec{y})$. In this approximation the interaction term $\lambda\phi^4$ becomes quadratic, i.e., it contributes to a self consistent mass. These equations were generalized for the out of thermo-dynamical equilibrium using different methods in [4, 7].

Using the Gaussian ansatz (3) in the symmetric phase ($\bar{\phi} = 0$) we need only two initial conditions for the temporal evolution of these equations, $G(t = 0)$ and $\dot{G}(t = 0)$, which is proportional to its imaginary part Σ . An inspection of the equations of motion shows that for $\bar{\phi}(t = 0) = \dot{\bar{\phi}}(t = 0) = 0$ the variables $\bar{\phi}$ and $\bar{\pi}$ will not change, i.e., the classical part of the field will remain constant (zero) during all times in the frame of this approach.

Several numerical works have been done for the above equations of movement. The choice of initial conditions is entirely subordinate to the Gaussian approximation. If it were not Gaussian, we would have to consider three conditions instead of two [15]. The analysis of these equations by Boyanovsky *et al* and by [9] shows that initial conditions (for homogeneous G and $\bar{\phi}$) are crucial for the time interval in which the system evolves towards the minimum and for the speed of the field evolution.

3 Numerical method

The numerical method for the temporal evolution of the initial conditions corresponding to the system described above used in this paper was developed in [9, 16]. This section is a brief review. One defines a generalized density matrix in a lattice as:

$$R_{i,j} = \begin{pmatrix} \rho_{i,j} & \kappa_{i,j} \\ -\kappa_{i,j}^* & -\rho_{i,j}^* \end{pmatrix}, \quad (8)$$

where the density matrices (mean values) are given by: $\rho_{i,j} = \frac{1}{2} \langle a_i a_j^\dagger + a_i^\dagger a_j \rangle$ which is hermitian and $\kappa_{i,j} = - \langle a_i a_j \rangle$ is symmetric, using the creation and annihilation operators. These operators can be written in a lattice with mesh size Δx in d spatial dimensions as:

$$a(j) = \frac{1}{\sqrt{2}} \left\{ \phi(j)(\Delta x)^{\frac{d-1}{2}} + i\pi(j)(\Delta x)^{\frac{1+d}{2}} \right\} \quad (9)$$

$$a^\dagger(j) = \frac{1}{\sqrt{2}} \left\{ \phi(j)(\Delta x)^{\frac{d-1}{2}} - i\pi(j)(\Delta x)^{\frac{1+d}{2}} \right\} \quad (10)$$

Some of the calculated mean values in terms of the matrix elements of the above matrix are given by:

$$\begin{aligned} G(i, j) &= \langle \phi(i)\phi(j) \rangle = \frac{1}{(\Delta x)^{d-1}} \Re(\rho(i, j) - \kappa(i, j)) \\ F(i, j) &= \langle \Pi(i)\Pi(j) \rangle = \frac{1}{(\Delta x)^{d+1}} \Re(\rho(i, j) + \kappa(i, j)) \end{aligned} \quad (11)$$

where $F(i, j) = G(i, j)^{-1}/4 + 4\Sigma(i, k)G(k, l)\Sigma(l, i)$.

The temporal evolution is governed by the Hartree (Fock) Bogoliubov energy, which can be parametrized in the following form:

$$\frac{1}{2}H_{i,j} = \frac{\delta E}{\delta R_{j,i}} = \begin{pmatrix} W_{i,j} & D_{i,j} \\ -D_{i,j} & -W_{i,j} \end{pmatrix}, \quad (12)$$

where the above matrices are given in terms of the parameters of the model [9]:

$$W_{i,j} = \frac{1}{2}(-\Delta_{i,j} + (m_0^2 + \lambda G_{i,i}/2 + (\Delta x)^{-2})\delta_{i,j}), \quad (13)$$

$$D_{i,j} = \frac{1}{2}(-\Delta_{i,j} + (m_0^2 + \lambda G_{i,i}/2 - (\Delta x)^{-2})\delta_{i,j}). \quad (14)$$

With these matrices, one can check that the Liouville-von Neumann condition is satisfied:

$$i\dot{R}_{ij} = [H_{ik}, R_{kj}]. \quad (15)$$

These equations are equivalent to those obtained from the Gaussian approximation of the previous section [9]. The time-evolution of the generalized density matrix can be then performed for given initial conditions.

4 Numerical results

In this section we will report results of numerical calculations for the equations of motion studied above in a lattice of 100 points with spacing $\Delta x = 0.1 fm$. By varying $\Delta x \rightarrow 0$ it was possible to assure the reliability of the results (dynamics does not change in this limit). The lattice spacing is always to be much smaller than the correlation length ($\xi = 1/m_{phys}$) which is quite smaller than the lattice size (10 fm). The first initial condition for the classical part of the field is given by:

$$\bar{\phi}(t=0) = \bar{\phi}_0 \tanh^2\left(\frac{x-x_0}{0.5}\right), \quad \dot{\bar{\phi}} = \bar{\pi} = 0, \quad (16)$$

where x_0 is the center of the lattice and the coordinate x is discretized. This configuration is not stable since in a small region of the lattice the field has values different from the vacuum.

The field configuration corresponding to (16) is shown in Figure 1 with a solid line for the model with a strong coupling constant $\lambda = 600 fm^{-2}$ and physical mass $\mu = 100 MeV$. In terms of energy it corresponds to a bubble of high energy density in the vacuum which can be interpreted as an *in medium* effect. This initial condition is plugged into the system of equations (7) and numerically evolved with the method described in the last section. Knowing the field configuration at all times we can compute the energy density distribution at all time steps for different scenarios. Periodic boundary conditions are considered in this work.

In Figure 2a we show the energy density distribution of the classical part of the field alone (which we call ρ) along the x direction for different times $t = 0$ (thick solid), $= 0.1$ (solid), $= 1$ (thin solid), 2 (thick dotted) and 3.5 (dotted) fm. As it can be seen, the energy density excess with relation to the vacuum value is distributed among the lattice by means of “waves”. The expansion of the initially high energy density proceeds with two peaks for each side of the lattice. This feature may be expected for the massless Klein-Gordon equation which has a sinoidal solution due to the gradient terms. The energy density is proportional to the square of the field and consequently we obtain this “two bumps” structure.

In Figure 2b the same initial conditions for the condensate are evolved in time with the contribution from the quantum fluctuations, i.e., considering all the four equations (7). For the quantum part along this work, it was assumed that, at the initial time, they are at the vacuum value, i.e.:

$$G(r, t = 0) = G_0 \quad \Sigma \propto \dot{G}(r, t = 0) = 0, \quad (17)$$

where G_0 is the value of the quantum fluctuations in the vacuum of the asymmetric phase ($\bar{\phi} \neq 0$) obtained from a gap equation. In this case the gap equation is given by:

$$\mu^2 = m_0^2 + \frac{\lambda}{2} G_0(\mu^2) + \frac{\lambda}{2} \bar{\phi}_0^2 \quad (18)$$

The value of G_0 can be fixed by the physical mass μ . There is, now, energy transfer from the classical part to the quantum fluctuations and back. Comparing figures 2a and 2b we can clearly see that the inclusion of quantum fluctuations accelerates the expansion. This is especially visible at $t = 3.5$ fm and even more at higher times. The effect is small but unambiguous. We observe an inversion in the heights of the two bumps that characterize the energy distribution, the outer bump becoming more pronounced in the quantum case. This is an indication that, quantum fluctuations increase the pressure in spite of the isoentropic character of the approximation. Furthermore, the energy density configuration is clearly smoother than in the purely classical case. It is worth to remember once more that the present approach is not yet completely “self-consistent” in the sense that for given initial “non-equilibrium” conditions the model should reflect this fact including, for instance, the coupling constants and quantum fluctuations. This is under study [14].

In order to investigate the sensitivity of the conclusion found in the last paragraph to the shape of the initial conditions, we repeat the same calculations starting from the following field profile:

$$\bar{\phi}(t = 0) = \bar{\phi}_0 \left\{ 1 + 0.3 \operatorname{sech}^2 \left(\frac{x - x_0}{0.5} \right) \right\}. \quad (19)$$

The corresponding field configuration is shown in Figure 1 with a dashed line. In this case we use a different coupling constant $\lambda = 60 fm^{-2}$. This choice changes the global normalization of the field and energy,

but it does not change the dynamics considerably. As a matter of fact, in the classical level, there is a scale invariance in the dynamics with relation to changes in such variable. Taking into account quantum fluctuations this scale invariance is broken (slightly for the range of parameters considered in this work). Evolving only the classical part of the field in the same way as before we obtain Figure 3a, which is very similar to Fig. 2a. Switching on the quantum fluctuations leads to results of Fig. 3b. Comparing Figs. 3a and 3b we observe the same effect already seen in Fig. 2, i.e., the inclusion of quantum fluctuations accelerates the expansion.

In the asymmetric phase the effective potential has a Mexican hat like form and in all cases presented in this work the total energy is smaller than the “barrier” (located at the origin of the $\bar{\phi}$ space) which separates the two minima of the potential. In the cases shown in Figures 2b and 3b there is a tunneling of the condensate from one side of the potential to the other. For some kinds of initial condition this was already shown in [11]. This effect disappears if the quantum fluctuations are switched off but remain when the lattice spacing is reduced as discussed below.

The presence of quantum fluctuations make the energy density to be amplified in the earlier times and to expand faster. This amplification is stronger in the case shown in figure 3b. As time goes the energy density “waves” tend to decrease. Moreover, we notice small regions with energy density lower than the true vacuum of the corresponding phase in the beginning of the expansion, just after the energy density amplification happens.

The total energy of the system is conserved during the evolution. We want to stress that the inclusion of quantum fluctuations changes the overall energy normalization of the system. Therefore, the initial energy profile is similar for all figures but the normalization is different as can be seen in the pictures.

The equations of movement (7) exhibit Ultra Violet (UV) divergences in a continuum space-time which are exactly the same as those present in the GAP equation for the vacuum of the model. They require mass renormalization (and also coupling constant renormalization depending on the space-time dimension) which is exactly the same for the static and time dependent cases. In [9, 11] analytical solutions for these equations of movement were found for a particular kind of initial conditions. They exhibit the UV divergences only at the initial time ($t=0$). As this divergence is eliminated by the renormalization (of the mass and/or coupling) the temporal evolution remains unchanged. This is also true in the lattice: the deviation of the system with relation to the vacuum state determines the temporal evolution. For a variation of the lattice spacing, i.e., as $\Delta x \rightarrow 0$ (for low values as 0.02 fm were checked) the dynamics does not change considerably (with the corresponding mass rescaling). In fact, the continuum limit must be the same for the static and

time-dependent cases. The deviations from this (renormalized) state are always finite and are not altered by vacuum redefinition (renormalization). Mass renormalization changes the normalization of the energy density being, therefore, not important for the dynamical evolution.

In fact, the difference between the classical system and the classical plus quantum system is, in part, related to a scale invariance. The classical level exhibits scale invariance but this is not present at the quantum level. These effects will be explicitly addressed in detail elsewhere [14].

Our results point out to the following simple picture: without quantum fluctuations a given localized spatial configuration of the field $\bar{\phi}$ expands, simply converting “potential” into “kinetic” energy. The inclusion of these fluctuations accelerates the expansion. This effect may be caused by the interaction between the particles and the condensate, by the tunneling between the two vacua or by these both aspects. The questions addressed in this work are related to those discussed in Ref. [1]. In that work, the expansion was driven by the effective potential, (which contains a $\lambda\phi^4$ part) plus the contribution of a gas of pions. The former and latter contributions correspond respectively to our classical and quantum contributions. The total pressure is the sum of both the effective potential and the pion gas component and therefore also in that model the inclusion of quantum fluctuations increases the total pressure and the expansion rate. These conclusions also are in agreement with studies for the effects of quantum fluctuations in the Inflationary scenario where there is “acceleration” of the dynamics, see for example [10, 17].

5 Summary

We have analyzed the temporal evolution of non-homogeneous configurations of the $\lambda\phi^4$ model considering two different approaches: the classical equations of motion and the equations of motion in the framework of the Gaussian approach for the quantum fluctuations in a one-dimensional lattice. We have been able to study the influence of the quantum fluctuations on the classical field dynamics and vice versa. Quantum fluctuations make the energy distribution smoother. Besides that, we conclude that the quantum fluctuations accelerate the expansion of a non homogeneous configuration of the classical part of the field after having amplified the energy density. We noted a tunneling of the condensate from one side of the “Mexican-hat” potential to the other when quantum fluctuations are considered depending on the initial conditions.

We would like to emphasize that in the limit of vanishing lattice spacing the dynamics is not qualitatively modified: the renormalization acts as to change the field and therefore energy density normalization but not the dynamics. We have been concerned mainly with short interval evolution, but the evolution of

thermal degrees of freedom will be analyzed elsewhere. Finally with the inclusion of temperature, we hope to establish a closer connection between our system and a hydro-dynamically expanding fireball.

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Figure Captions

Fig. 1 Initial field configuration. Solid line corresponds to (16) and the dashed line line to (19).

Fig. 2 a) Evolution of the energy distribution of the condensate at different times; b) the same as a) with the inclusion of quantum fluctuations. The initial field configuration is given by (16).

Fig. 3 The same as Fig.2 for the initial configuration (19).